Distributioner på TI-89

Home -> Catalog -> F3 -> Funktionsnavn

	Værdi -> Sandsynlighed P	Sandsynlighed P -> Værdi
Chi-square	chi2cdf(0, værdi, DF)	invChi2(P, DF)
t distribution	1 - tCdf(værdi, 10^99, DF)	inv_t(P, DF)
F distribution	fCdf(0, værdi, DFnum, DFdenum)	invF(P, DFnum, DFdenum)

Opgave 1

Generelle ting

$$S = \sum_{i=1}^{n} x_i$$

$$\bar{x}. = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n} S \sim\sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}.\right)^2 = \frac{1}{n-1} SSD \sim\sim \sigma^2 \chi^2(f) / f$$

$$SSD = \sum_{i=1}^{n}\left(x_i - \bar{x}.\right)^2 = USS - \frac{S^2}{n}$$

$$USS = \sum_{i=1}^{n} x_i^2$$

1: k = 2 normalfordelte observationsrækker

1.1: Model $M : x_{ij} \sim\sim N\left(\mu_i, \sigma_i^2\right)\ i = 1,2 \quad j = 1,\ldots,n_i$

Frihedsgrader: $f_{(1)} = n_1 - 1, \quad f_{(2)} = n_2 - 1$

Estimater

$$\mu_1 \leftarrow \bar{x}_1. \sim\sim N\left(\mu_1, \frac{\sigma_1^2}{n_1}\right), \qquad \sigma_1^2 \leftarrow s_{(1)}^2 \sim\sim \sigma_1^2 \chi^2\left(f_{(1)}\right) / f_{(1)}$$

$$\mu_2 \leftarrow \bar{x}_2. \sim\sim N\left(\mu_2, \frac{\sigma_2^2}{n_2}\right), \qquad \sigma_2^2 \leftarrow s_{(2)}^2 \sim\sim \sigma_2^2 \chi^2\left(f_{(2)}\right) / f_{(2)}$$

1.1.1: Test for $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (side 94, brug tabel s. 99 til beregninger☺):

Til model: $M : x_{ij} \sim\sim N\left(\mu_i, \sigma^2\right)\ i = 1,2 \quad j = 1,\ldots,n_i$

Teststørrelse:

$$s^2_{numerator} = \max\left\{s^2_{(1)}, s^2_{(2)}\right\}$$

$$s^2_{demoninator} = \min\left\{s^2_{(1)}, s^2_{(2)}\right\}$$

$$F = \frac{s^2_{numerator}}{s^2_{demoninator}}$$

Testsandsynlighed

$$f_{\text{numerator}} = \text{ frihedsgrader af } s^2_{numerator}$$

$$f_{\text{denominator}} = \text{ frihedsgrader af } s^2_{demoninator}$$

$$p_{obs}(x) = 2\left[1 - F_{F(f_{\text{numerator}}, f_{\text{denominator}})}(F)\right]$$

1.1.2: Test for $H_{0\mu}: \mu_1 = \mu_2 = \mu$ (side 95, brug tabel s. 99 til beregninger☺)

Teststørrelse

$$t(x) = \frac{\bar{x}_{1\cdot} - \bar{x}_{2\cdot}}{\sqrt{\frac{s^2_{(1)}}{n_1} + \frac{s^2_{(2)}}{n_2}}} \sim\sim t(\bar{f})$$

$$\bar{f} = \frac{\left(\frac{s^2_{(1)}}{n_1} + \frac{s^2_{(2)}}{n_2}\right)^2}{\frac{\left(\frac{s^2_{(1)}}{n_1}\right)^2}{f_{(1)}} + \frac{\left(\frac{s^2_{(2)}}{n_2}\right)^2}{f_{(2)}}}$$

Testsandsynlighed

$$p_{obs}(x) = 2\left[1 - F_{t(\bar{f})}\left(\left|t(x)\right|\right)\right]$$

1.1.3: $(1-\alpha)$ konfidensinterval for σ_i^2 (beregnes som for 1 observationsrække – side 78)

$$\left[\frac{f_{(i)} s^2_{(i)}}{\chi^2_{1-\alpha/2}\left(f_{(i)}\right)} \le \sigma_i^2 \le \frac{f_{(i)} s^2_{(i)}}{\chi^2_{\alpha/2}\left(f_{(i)}\right)}\right]$$

1.2: Model $M: x_{ij} \sim\sim N\left(\mu_i, \sigma^2\right)\ i=1,2 \quad j=1,\ldots,n_i$

Frihedsgrader: $f_{(1)} = n_1 - 1, \quad f_{(2)} = n_2 - 1$

Estimater

$$\mu_1 \leftarrow \bar{x}_{1\cdot} \sim\sim N\left(\mu_1, \frac{\sigma^2}{n_1}\right)$$

$$\mu_2 \leftarrow \bar{x}_{2\cdot} \sim\sim N\left(\mu_2, \frac{\sigma^2}{n_2}\right)$$

$$\sigma^2 \leftarrow s_1^2 = \frac{f_{(1)}s_{(1)}^2 + f_{(2)}s_{(2)}^2}{f_{(1)} + f_{(2)}} = \frac{SSD_{(1)} + SSD_{(2)}}{f_1} \sim\sim \sigma^2\chi^2\left(f_1\right)/f_1, \quad f_1 = f_{(1)} + f_{(2)} = n_1 + n_2 - 2$$

1.2.1: Test for $H_{0\mu}: \mu_1 = \mu_2$ (side 94, brug tabel s. 99 til beregninger☺*)*

Til model: $M: x_{ij} \sim\sim N\left(\mu, \sigma^2\right)\ i=1,2 \quad j=1,\ldots,n_i$

Teststørrelse

$$t(x) = \frac{\bar{x}_{1\cdot} - \bar{x}_{2\cdot}}{\sqrt{s_1^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim\sim t\left(f_1\right)$$

Testsandsynlighed

$$p_{obs}(x) = 2\left[1 - F_{t(f_1)}\left(\left|t(x)\right|\right)\right]$$

1.2.1: Test for $H_0: \sigma^2 = \sigma_0^2$ (bestemt værdi af varians – side 78)

Til model: $M: x_{ij} \sim\sim N\left(\mu_i, \sigma_0^2\right)\ i=1,2 \quad j=1,\ldots,n_i$

Teststørrelse:

$$T = \frac{f_1 s_1^2}{\sigma_0^2}$$

Testsandsynlighed:

$$p_{obs}(x) = \begin{cases} 2F_{\chi^2(f_1)}(T) & hvis\ T \le f_1 \\ 2\left(1 - F_{\chi^2(f_1)}(T)\right) & ellers \end{cases}$$

1.2.2: $(1-\alpha)$ konfidensinterval for $\mu_1 - \mu_2$

$$\left\{\bar{x}_{1\cdot} - \bar{x}_{2\cdot} - \sqrt{s_1^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}t_{1-\alpha/2}(f_1) \le \mu_1 - \mu_2 \le \bar{x}_{1\cdot} - \bar{x}_{2\cdot} + \sqrt{s_1^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}t_{1-\alpha/2}(f_1)\right\}$$

1.2.2: $(1-\alpha)$ konfidensinterval for μ_i (side 78)

$$\left\{\bar{x}_{i\cdot} - \sqrt{\frac{s_1^2}{n_i}}t_{1-\alpha/2}(f_1) \le \mu_i \le \bar{x}_{i\cdot} + \sqrt{\frac{s_1^2}{n_i}}t_{1-\alpha/2}(f_1)\right\}$$

1.2.3: $(1-\alpha)$ konfidensinterval for den fælles varians σ^2 (beregnes som for 1 fordeling side 79)

$$\left\{\frac{f_1 s_1^2}{\chi^2_{1-\alpha/2}(f_1)} \le \sigma^2 \le \frac{f_1 s_1^2}{\chi^2_{\alpha/2}(f_1)}\right\}$$

1.3: Model $M_2 : x_j \sim\sim N(\mu, \sigma^2) \quad j = 1, \ldots, n_i$

Frihedsgrader: $f_2 = n_1 + n_2 - 1$

Estimater

$$\mu \leftarrow \bar{x}_{\cdot\cdot} = \frac{S_{(1)} + S_{(2)}}{n_1 + n_2}$$

$$\sigma^2 \leftarrow s_{02}^2 = \frac{1}{f_2} SSD_{02} = \frac{1}{f_2}\left(\left(USS_{(1)} + USS_{(2)}\right) - \frac{\left(S_{(1)} + S_{(2)}\right)^2}{n_1 + n_2}\right) \sim\sim \sigma^2 \chi^2(f_2)$$

1.3.1: $(1-\alpha)$ konfidensinterval for den fælles middelværdi (beregnes som for 1 fordeling side 79)

$$\left\{\bar{x}_{\cdot\cdot} - \sqrt{\frac{s_{02}^2}{n_1 + n_2}}t_{1-\alpha/2}(f_2) \le \mu \le \bar{x}_{\cdot\cdot} + \sqrt{\frac{s_{02}^2}{n_1 + n_2}}t_{1-\alpha/2}(f_2)\right\}$$

Opgave 2

2.1: Likelihood funktionen $L(\mu)$ (side 71)

Tæthedsfunktion for $N\left(\mu,\sigma^2\right)$:

$$f_X = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

Likelihood funktionen af givet ved $L(\mu) = \prod_{i=1}^{n} f_{X_i}(x)$

Regler

$$e^a e^b = e^{a+b}$$

$$\sqrt{x}^a = x^{\frac{a}{2}}$$

$$a^b c^b = (ac)^b$$

2.2: Maksimum likelihood estimat

Vi finder log-likelihood funktionen:

$$l(\mu) = \ln(L(\mu))$$

Regler:

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(a \cdot b) = \ln(a) + \ln(b)$$

$$\ln\left(a^b\right) = b \cdot \ln(a)$$

$\frac{d}{dx}u^n = nu^{n-1}\frac{d}{dx}u$ // Power rule

$(f \circ g)' = (f' \circ g)g'$ // Chain rule

2.3: Fordeling og konfidensinterval

2.3.1: Vis at .... er normalfordelt med .... parametre: vis at ... ~~ $N\left(\mu,\sigma^2\right)$ (side 161)

Brug (3.81):

$$Y = c_0 + c_1 X_1 + \ldots + c_n X_n$$

$$Y \sim N\left(c_0 + c_1\mu_1 + \ldots + c_n\mu_n,\ c_1^2\sigma_1^2 + \ldots + c_n^2\sigma_n^2\right)$$

2.3.2: Gør rede for at $\sigma^{2} \sim\sim \sigma^2 \chi^2(\ldots)/\ldots$

Brug hovedsagligt: 160-163

Regler

$$X \sim N(\mu, \sigma^2) \Leftrightarrow \frac{X-\mu}{\sigma} \sim N(0,1) \quad // \ (3.83)$$

$$U \sim N(0,1) \Rightarrow U^2 \sim \chi^2(1) \quad // \ (3.92)$$

$$\sum_i^n (X_i - \mu)^2 \sim \sigma^2 \chi^2(n) = \sum_i^n \frac{(X_i-\mu)^2}{\sigma^2} \sim \chi^2(n) \quad // \ (3.93)$$

2.3.3: Udled en formel for et 95% konfidensinterval for σ^2

Vi har maksmimum likelihood estimatet af σ^2 som er: $\sigma^{2} \sim\sim \sigma^2\chi^2(f)/(f)$.

Iflg. s. 61:

$s^2(X)/\sigma^2 \ \ is \ \ \chi^2(f)/f \ \ distributed$ (hvor i vores tilfælde; $s^2(X) = \sigma^{2}$)

1. Vi tjekker om vi må bruge formlen ved at dividere med σ^2 og tjekker om vi får samme distribution

$$\frac{\sigma^{2}}{\sigma^2} \sim\sim \chi^2(f)/(f)?$$

2. Hvis det passer så gælder der at:

$$1-a = P\left[\frac{\chi^2_{\alpha/2}(f)}{f} \le \frac{s^2(X)}{\sigma^2} \le \frac{\chi^2_{1-\alpha/2}(f)}{f}\right]$$

$$= P\left[\frac{fs^2(X)}{\chi^2_{1-\alpha/2}(f)} \le \sigma^2 \le \frac{fs^2(X)}{\chi^2_{\alpha/2}(f)}\right]$$

Hvor følgende så er konfidensintervallet:

$$\left[\frac{fs^2}{\chi^2_{1-\alpha/2}(f)}, \quad \frac{fs^2}{\chi^2_{\alpha/2}(f)}\right]$$

Opgave 3

Til-og-fra formlen (side 187)

Main Points in Section 4.1

The salient feature of linear normal models is the ease with which a reducution from one linear normal model,

$$M_1 : \boldsymbol{\mu} \in L_1,$$

to a simpler linear normal model,

$$M_2 : \boldsymbol{\mu} \in L_2,$$

where simpler means that L_2 is included in L_1.

Test of the reduction $M_1 \to M_2$:

Test statistic

$$F = \frac{\dfrac{\|\mathbf{x} - P_2(\mathbf{x})\|^2 - \|\mathbf{x} - P_1(\mathbf{x})\|^2}{(n - d_2) - (n - d_1)}}{s_{01}^2} \sim\sim F(d_1 - d_2, n - d_1)$$

with significance probability

$$p_{obs}(\mathbf{x}) = 1 - F_{F(d_1 - d_2,\, n - d_1)}(F(\mathbf{x})).$$

The quantities needed to calculate the F-test are found in the ERROR lines of the analysis of variance tables of the two models as indicated in the tables below.

Table 4.3 *The analysis of variance table from model* M_1

The GLM Procedure
Model M_1

Dependent Variable: X

	Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
	Model	$d_1 - 1$	$\|P_1(\mathbf{x}) - P_\infty(\mathbf{x})\|^2$	$s_{1\infty}^2$	$s_{1\infty}^2 / s_{01}^2$	$p_{obs}(\mathbf{x})$
→	Error	$n - d_1$	$\|\mathbf{x} - P_1(\mathbf{x})\|^2$	s_{01}^2		
	Corrected Total	$n - 1$	$\|\mathbf{x} - P_\infty(\mathbf{x})\|^2$			

Table 4.4 *The analysis of variance table from model* M_2

The GLM Procedure
Model M_2

Dependent Variable: X

	Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
	Model	$d_2 - 1$	$\|P_2(\mathbf{x}) - P_\infty(\mathbf{x})\|^2$	$s_{2\infty}^2$	$s_{2\infty}^2 / s_{02}^2$	$p_{obs}(\mathbf{x})$
→	Error	$n - d_2$	$\|\mathbf{x} - P_2(\mathbf{x})\|^2$	s_{02}^2		
	Corrected Total	$n - 1$	$\|\mathbf{x} - P_\infty(\mathbf{x})\|^2$			

3.1: Model $M : x_{ij} \sim\sim N\left(\alpha + \beta t_i, \sigma^2\right) \quad i = 1,\dots,k \quad j = 1,..,n_i$

$f_{02} = n - 2$

3.1.1: Test $H_0 : \alpha = \alpha_0$

Aflæsning i SAS (*kun for test a = 0) (side 145)*

Teststørrelse for a = 0 kan aflæses i SAS output for M under Parameter i "Intercept" linjen ved "t Value".

Testsandsynlighed for a = 0 kan aflæses i SAS output for M under Parameter i "Intercept" linjen ved "Pr > |t|".

Beregning (side 157)

Teststørrelse

$$t(x) = \frac{\alpha - \alpha_0}{\sqrt{s_{02}^2\left(\frac{1}{n.} + \frac{\bar{t.}^2}{SSD_t}\right)}} \sim\sim t(n. - 2)$$

Testsandsynlighed

$$p_{obs}(x) = 2\left[1 - F_{t(n.-2)}\left(\left|t(x)\right|\right)\right]$$

3.1.2: Test for $H_0 : \beta = \beta_0$

Aflæsning i SAS (*kun for test b = 0) (side 145)*

Teststørrelse for b = 0 kan aflæses i SAS output for M under Parameter i "navn på beta" linjen ved "t Value".

Testsandsynlighed for b = 0 kan aflæses i SAS output for M under Parameter i "navn på data" linjen ved "Pr > |t|".

Beregning (side 157)

Teststørrelse

$$t(x) = \frac{\beta - \beta_0}{\sqrt{s_{02}^2 / SSD_t}} \sim\sim t(n. - 2)$$

Testsandsynlighed

$$p_{obs}(x) = 2\left[1 - F_{t(n.-2)}\left(\left|t(x)\right|\right)\right]$$

3.1.3: $(1-\alpha)$ konfidensinterval for α

Aflæsning i SAS (side 145)

Frihedsgraderne f_{02} *aflæses i SAS output for M under Error linjen ved "DF".*

Aflæses i Paremeter linjen:

$Estimate \pm StdError \cdot t_{1-\alpha/2}\left(f_{02}\right)$ *hvor* f_{02} *er frihedsgraderne for det brugte variansskøn*

Beregning (side 155)

$$\left[\alpha-\sqrt{s_{02}^2\left(\frac{1}{n.}+\frac{\bar{t}.^2}{SSD_t}\right)}t_{1-\alpha/2}\left(f_{02}\right),\ \alpha+\sqrt{s_{02}^2\left(\frac{1}{n.}+\frac{\bar{t}.^2}{SSD_t}\right)}t_{1-\alpha/2}\left(f_{02}\right)\right]$$

3.1.4: $(1-\alpha)$ konfidensinterval for β

Aflæsning i SAS (side 145)

Frihedsgraderne f_{02} *aflæses i SAS output for M under Error linjen ved "DF".*

Aflæses i Paremeter linjen:

$Estimate \pm StdError \cdot t_{1-\alpha/2}\left(f_{02}\right)$ *hvor* f_{02} *er frihedsgraderne for det brugte variansskøn*

Beregning (side 155)

$$\left[\beta-\sqrt{\frac{s_{02}^2}{SSD_t}}t_{1-\alpha/2}\left(f_{02}\right),\ \beta+\sqrt{\frac{s_{02}^2}{SSD_t}}t_{1-\alpha/2}\left(f_{02}\right)\right]$$