Middelværdi

$EX = \sum_{x \in \text{supp } p_x} x \cdot p_x(x)$ Definition 6.1 side 63

$EX^2 = \sum_{x \in \text{supp } p_x} x^2 \cdot p_x(x)$ 6.1 side 65

$EX = \int_{-\infty}^{\infty} x \cdot f_x(x)\, dx$ 6.2' side 66

$EX^2 = \int_{-\infty}^{\infty} x^2 \cdot f_x(x)\, dx$ 6.2 side 65

$E(X \cdot Y) = EX \cdot EY$ hvis X og Y er uafh. 6.16 (theorem 6.7) side 69

$\mathrm{E}(X^2 \cdot Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^2 \cdot y \cdot f_{x,y}(x,y)\, dx dy$ 6.4' side 66

$\mathrm{E}(X+Y) = E(X) + E(Y)$ 6.7 side 67

Varians og covarians (scan: side 72)

Theorem 6.10 For three random variables X, Y and Z and for constants $a, b \in \mathbb{R}$ we have

$$\operatorname{Var} X = EX^2 - (EX)^2, \tag{6.20}$$

$$\operatorname{Var}(a + bX) = b^2 \operatorname{Var} X, \tag{6.21}$$

$$\operatorname{Var}(X+Y) = \operatorname{Var} X + \operatorname{Var} Y + 2\operatorname{Cov}(X,Y), \tag{6.22}$$

$$\operatorname{Cov}(X,Y) = E(XY) - EXEY, \tag{6.23}$$

$$-1 \le \operatorname{Cor}(X,Y) \le 1, \tag{6.24}$$

$$\operatorname{Cov}(aX, bY) = ab\operatorname{Cov}(X,Y) \tag{6.25}$$

$$\operatorname{Cov}(X, Y+Z) = \operatorname{Cov}(X,Y) + \operatorname{Cov}(X,Z). \tag{6.26}$$

Furthermore, if X and Y are *independent*, then

$$\operatorname{Cov}(X,Y) = \operatorname{Cor}(X,Y) = 0, \tag{6.27}$$

i.e., X and Y are uncorrelated.

If X and Y are uncorrelated, in particular, if X and Y are independent, we have, moreover, that

$$\operatorname{Var}(X+Y) = \operatorname{Var} X + \operatorname{Var} Y \tag{6.28}$$

Finally, if $\mathbf{X}$ is a k-dimensional random vector, $\mathbf{a}$ is a d-dimensional vector and $\mathbf{B}$ is a $k \times d$ matrix, then the covariance matrix of the d-dimensional random vector $\mathbf{a} + \mathbf{XB}$ is

$$\mathbf{Cov}(\mathbf{a} + \mathbf{XB}) = \mathbf{B}^*(\mathbf{Cov}\,\mathbf{X})\mathbf{B}, \tag{6.29}$$

and if $\mathbf{X}$ and $\mathbf{Y}$ are independent k-dimensional random vectors, then

$$\mathbf{Cov}(\mathbf{X} + \mathbf{Y}) = \mathbf{Cov}(\mathbf{X}) + \mathbf{Cov}(\mathbf{Y}). \tag{6.30}$$

Tætheder

$$f_x(x) = \int_{-\infty}^{\infty} f_{x,y}(x,y)\,dy$$ 5.69 side 53

Distributioner

Theorem 5.32 side 44.

Theorem 5.32 Let X have distribution function F. Assume that F is continuous and, furthermore, that F' exists and is continuous in all but finitely many points $t_1 < t_2 < \cdots < t_n$. Then X is absolutely continuous with density f given by

$$f(x) = \begin{cases} F'(x) & \text{if } x \notin \{t_1, \ldots, t_n\} \\ 0 & \text{if } x \in \{t_1, \ldots, t_n\}. \end{cases} \tag{5.37}$$

♦

Sandsynligheder

$$P(X \in]a,b]) = F(b) - F(a) = \int_a^b f(z)\,dz \quad a < b$$ 5.32 side 43

$$P_x(x) = \sum_y P_{x,y}(x,y)$$ 5.18 side 32

Uafhængighed

$uafh.\,discrete?\ P_{x,y} \overset{?}{=} P_x P_y$ 5.75 side 57

$uafh.\,absolut?\ f_{x,y} = f_x f_y$ 5.74 side 56

Integraler

$$[x]_b^a = a - b$$

$$\int e^{ax} = \frac{1}{a} e^{ax}$$ se side 141 for andre

Middelværdier og varianser for kendte fordelinger (scan: side 78)

Distribution *of* X	*Mean* EX	*Variance (Covariance)* $\mathrm{Var}\,X$ $(\mathbf{Cov}\,\mathbf{X})$
$b(n,\pi)$	$n\pi$	$n\pi(1-\pi)$
$\mathrm{po}(\lambda)$	λ	λ
$b^-(\kappa,\pi)$	$\frac{\kappa\pi}{1-\pi}$	$\frac{\kappa\pi}{(1-\pi)^2}$
$h(M,N,n)$	$n\frac{M}{N}$	$n\frac{M}{N}\left(1-\frac{M}{N}\right)\frac{N-n}{N-1}$
$R(0,1)$	$\frac{1}{2}$	$\frac{1}{12}$
$N(\mu,\sigma^2)$	μ	σ^2
$\Gamma(\alpha,\lambda)$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$
$e(\lambda)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$\chi^2(f)$	f	$2f$
$m_k(n,\boldsymbol{\pi})$	$n\boldsymbol{\pi}$	$(\mathbf{Cov}\,\mathbf{X})_{ij} = \begin{cases} n\pi_i(1-\pi_i) & i=j \\ -n\pi_i\pi_j & i\neq j \end{cases}$
$N_k(\boldsymbol{\mu},\boldsymbol{\Sigma})$	$\boldsymbol{\mu}$	$\mathbf{Cov}\,\mathbf{X} = \boldsymbol{\Sigma}$

Table 6.1: Mean and variance of some of the standard distributions.

Distributioner lagt sammen (scan: side 94)

Distribution of $X_j,\ j=1,2$	Distribution of $X_\cdot = X_1 + X_2$
$b(n_j,\pi)$	$b(n_1+n_2,\pi)$
$\mathrm{po}(\lambda_j)$	$\mathrm{po}(\lambda_1+\lambda_2)$
$b^-(\kappa_j,\pi)$	$b^-(\kappa_1+\kappa_2,\pi)$
$\Gamma(\alpha_j,\lambda)$	$\Gamma(\alpha_1+\alpha_2,\lambda)$
$e(\lambda)$	$\Gamma(2,\lambda)$
$\chi^2(f_j)$	$\chi^2(f_1+f_2)$
$N(\mu_j,\sigma_j^2)$	$N(\mu_1+\mu_2,\sigma_1^2+\sigma_2^2)$
$m_k(n_j,\boldsymbol{\pi})$	$m_k(n_1+n_2,\boldsymbol{\pi})$
$N_k(\boldsymbol{\mu}_j,\boldsymbol{\Sigma}_j)$	$N_k(\boldsymbol{\mu}_1+\boldsymbol{\mu}_2,\boldsymbol{\Sigma}_1+\boldsymbol{\Sigma}_2)$

Table 8.1: Distribution of the sum $X_\cdot = X_1 + X_2$, where X_1 and X_2 are independent, for some of the distributions in Chapter 5.

Kendte fordelinger

Poison: $X \sim po(\lambda) = p(x) = e^{-\lambda}\frac{\lambda^x}{x!} \quad x \in \{0,1,2,\ldots\}$ side 26

Binominal: $X \sim b(n,\pi) = p(x) = \binom{n}{x}\pi^x(1-\pi)^{n-x} = \frac{n!}{x!(n-x)!}\pi^x(1-\pi)^{n-x} \quad x \in \{0,1,\ldots,n\}$ side 25

Negative binomial: side 27

Geometric: side 27

Hypergeometric: side 28

Normal: $X \sim N(\mu,\sigma^2) = f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad x \in \mathbb{R}$ side 111

Standard normal: normal med parameter (0, 1) = $N(0,1)$